

Qur'an and Numeral System

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Before the advent of Islam, neither mathematics, nor numerals were present in proper scientific form. Even the shapes or pictures used to denote numerals were not proper. The base and place value concepts were also deficient. There was not proper, systematic and continuous number system before Islam. Only simple arithmetic and counting was present in rough form. Even the operations of addition subtraction were not possible. Abacus or tables were used for addition and subtraction and this method was so complex and unscientific in nature. Hence the absence of number system remained hindrance in the development of mathematics for hundreds of years.

It was the kindness of Quran which appreciated the knowledge, developed human thinking and method of thinking and it also compelled the human intellect for scientific knowledge/mathematics. Islam attributed determinations of direction of Qibla, time of salat, calculation of calendar for Haj, and Ramdan with mathematical calculation. This inspiration and compulsion by Quran for sciences resulted into invention of numeral and mathematics.

Mohammad Bin Musa Al-Khawarizmi was the first mathematician in the history who wrote three books on the subject and invented geometric shaped numerals, with concept of place value and using 10 and 60 as base value for his numeral systems. He also invented zero. But all the writer of history of mathematics like Carl B. Boyer, E. T. Bell, David Eugene Smith and Florian Cajori etc tried to avoid or minimize or discredit or neglect the scale of muslim contribution to mathematics by one way or the other in their authorships. Even our youth is unaware of the greatest achievements of early muslim scientists.

So to throw the light on the historical facts, we have wrote this paper. Mathematics is a branch of knowledge so we have given in beginning the Quranic attitude towards knowledge. Then we have described Quranic orders of making observation, thinking, reasoning and inferring result which became the basis of scientific method. Then we have given the shapes of numerals used by early civilizations with some conceptual discussion. Although presentation of pictures or shapes seems awkward but some times picture or shapes speak louder than words. Lastly I have given the numerals and number systems invented by the Muslim scientists Al-Khawarizmi, etc for comparison of readers to find out the invention of number system.

The Islamic civilization contributed a lot to formation of scientific method and invention of sciences. The Quranic inducement for mathematical sciences as well as Quranic mathematical concepts became main source of inspiration for invention of mathematical science by early Muslim scientists. The Quran also orders its follower for making observation, thinking,

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anylises, utilization of vision, application of reasoning and reflections¹, and derivation of results. These Quranic orders change the human attitude toward universe as well as develop the way of thinking or method of thinking to the scientific level. Consequently, the Muslim scholars invented numeral and number system, equation, algebra, trigonometry, and arithmetic. But all the writers of history of science and mathematics, intentionally or deliberately, tried to decrease the credit of Muslim scholar or discredit the Muslim scholars by one way or the other like; neglecting the contribution of Muslims while writing history of mathematics or by attributing the inventions of Muslims to the Western or by deshaping the names of Muslim scholars or writing Muslims as Non-Muslims, or calling Muslim as non-originators or creators, or calling Muslims as followers of Greeks or Indians². The famous historian of Mathematics Carl B. Boyer, in his book "A history of mathematics"³, E. T. Bell, in his book, "Development of Mathematics"⁴, David Eugene Smith, in his "history of mathematics"⁵, Howard Eves in his "An introduction of the history of Mathematics"⁶ tried to avoid or neglect or minimize or disgrace the Muslim contributions in numeral system and mathematics. So we have analyse the situation to find out the real value or status of Muslim contribution to number system. The Mathematic is the branch of knowledge so firstly we shall discuss Quranic attitude towards knowledge, than mathematical concepts present in Quran and Quranic inducement for mathematics and finally comparison of other civilizations numeral systems with the numeral system invented by the Muslim scholars.

QURANIC ATTITUDE TOWARDS KNOWLEDGE AND THINKING

The religion of Islam is quite different to other religions in its base and configuration regarding its attitude to knowledge and thought. Islam laid its bases on knowledge, intellect and wisdom and rationality. The importance of knowledge and intellect in Islam can be well determined by the fact that the first Revelation of Qur'an is about acquisition of knowledge and science. As it is said in the Qur'an (al-'Alaq, 96:1-5):

اقْرَأْ بِاسْمِ رَبِّكَ الَّذِي خَلَقَ ۝ خَلَقَ الْإِنْسَانَ مِنْ عَلَقٍ ۝ اقْرَأْ وَرَبُّكَ الْأَكْرَمُ ۝ الَّذِي عَلَّمَ بِالْقَلَمِ ۝ عَلَّمَ الْإِنْسَانَ مَا لَمْ يَعْلَمْ ۝

(O Beloved!) Read (commencing) with the Name of Allah, Who has created (everything). He created man from a hanging mass (clinging) like a leech (in the mother's womb). Read and your Lord is Most Generous, Who taught man (reading and writing) by the pen, Who (besides that) taught man (all that) which he did not know.⁷

It is worth mentioning that faith (*i'tiqadat*), worships (*'ibadat*) and ethics (*akhlaqiat*) always avail priority in beginning of all the religions. Islam is also a complete religion. So it was quite logical that first Qur'anic revelation should be about faith (*i'tiqadat*) like *tawhid*, *risala*, *aakhira*, or about worships (*'ibadat*) like *salat*, *hajj*, *fasting*, *zakat*, or about ethics (*akhlaqiat*). But the first revelation is about knowledge, acquisition of knowledge, creation of man (zoology). The Islamic attitude towards intellect, knowledge and vision is quite obvious from this fact and reality.

Islam has not only established its base on knowledge, thought and rationality but also the concept of dignity of man and its superiority over other creations including angels, has been based on the knowledge and character. As the Holy Qur'an stated in *Al-Baqara*, 2:31:

وَعَلَّمَ آدَمَ الْأَسْمَاءَ كُلَّهَا ثُمَّ عَرَضَهُمْ عَلَى الْمَلَائِكَةِ فَقَالَ أَنْبِئُونِي بِأَسْمَاءِ هَؤُلَاءِ
 إِنْ كُنْتُمْ صَادِقِينَ ۖ قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ
 الْحَكِيمُ ۚ قَالَ يَا آدَمُ أَنْبِئْهُمْ بِأَسْمَائِهِمْ فَلَمَّا أَنْبَأَهُمْ بِأَسْمَائِهِمْ قَالَ أَلَمْ أَقُلْ لَكُمْ
 إِنِّي أَعْلَمُ غَيْبَ السَّمَاوَاتِ وَالْأَرْضِ وَأَعْلَمُ مَا تُبْدُونَ وَمَا كُنْتُمْ تَكْتُمُونَ ۚ وَإِذْ قُلْنَا
 لِلْمَلَائِكَةِ اسْجُدُوا لِآدَمَ فَسَجَدُوا إِلَّا إِبْلِيسَ أَبَى وَاسْتَكْبَرَ وَكَانَ مِنَ الْكَافِرِينَ ۝

And Allah taught Adam the names of all (things), and then set them before the angels and said: 'Tell Me the names of these things if you are true (in your assumption).' The angels (humbly) submitted: 'Glory to You, You are Holy (Free from every deficiency). We have no knowledge except that which You have taught us. Surely, You alone are All-Knowing, All-Wise.' Allah said: 'O Adam, (now) apprise them of the names of these things.' So when Adam had told them the names of those things, (Allah) said: 'Did I not tell you I know (all) the hidden realities of the heavens and the earth

and also know all that you disclose and all that you conceal?' And (also recall the time) when We commanded the angels: 'Prostrate yourselves before Adam.' Then they all prostrated themselves to Adam except Iblis (Satan). He refused and showed arrogance and (consequently) became one of the disbelievers.⁸

When angels were ordered by Almighty Allah to tell the names of objects, they could not reply. But Adam rightly and correctly told the names of the objects or more generally the knowledge of all the things. Resultantly the angels were ordered to prostrate before Adam. The word *Isma* is not only used to denote the names of things or objects but it refers and indicates to one distinct discipline of knowledge. For example, the *Isma* mathematic is not only refers to name to one discipline of science but it also contains all knowledge and concepts of mathematics.

Even the inferiority of man has been related to knowledge and thoughts in Qur'an (Al-Jumu'a, 62:5).

مَثَلُ الَّذِينَ حُمِّلُوا التَّوْرَةَ ثُمَّ لَمْ يَحْمِلُوهَا كَمَثَلِ الْحِمَارِ يَحْمِلُ أَسْفَارًا بِئْسَ مَثَلُ الْقَوْمِ الَّذِينَ كَذَّبُوا بِآيَاتِ اللَّهِ وَاللَّهُ لَا يَهْدِي الْقَوْمَ الظَّالِمِينَ

The case of those who were burdened with (the Commandments and teachings of) the Torah (but) then did not bear it (i.e. this Messenger was mentioned in it but they disbelieved in him) is like that of a donkey loaded with heavy books on its back. How evil is the example of those who have rejected Allah's Revelations! And Allah does not guide the wrongdoers.⁹

At another place, it is said in sura Al-A'raf, 7:176:

وَلَوْ شِئْنَا لَرَفَعْنَاهُ بِهَا وَلَكِنَّهُ أَخْلَدَ إِلَى الْأَرْضِ وَاتَّبَعَ هَوَاهُ فَمَثَلُهُ كَمَثَلِ الْكَلْبِ إِنْ تَحْمِلُ عَلَيْهِ يَلْهَثُ أَوْ تَتْرُكُهُ يَلْهَثُ ذَلِكَ مَثَلُ الْقَوْمِ الَّذِينَ كَذَّبُوا بِآيَاتِنَا فَاقْصُصِ الْقَصَصَ لَعَلَّهُمْ يَتَفَكَّرُونَ

And had We so willed We would have exalted him by means of (knowledge and implementation of) these (Revelations), but he (himself) tracked down to (the lowest levels of the) earthly life and became the follower of his lust. (Now) his example is that of a dog. If you treat him harshly, he will loll out his

*tongue or if you leave him alone, he will (still) loll out his tongue. This example is of those who deny Our Revelations. So relate these occurrences (to the people) so that they may contemplate.*¹⁰

This verse shows that exaltation of mankind is based on knowledge and contemplation, and those who deny revelations (the most accurate knowledge) and contemplation or like dog and are tracked down to earthly life.

The Quran has not only associated the dignity/superiority of man and inferiority of man with level of knowledge and contemplation of but the Qur'an has much emphasized on knowledge, thinking, observation and contemplation. The Quranic verses about making of observation, contemplation, thinking, utilization of vision and wisdom and extraction of solution and conclusion of results lead to develop the human thinking to scientific level. These Quranic orders occurred repeatedly like the word *unzur* (أَنْظُرْ) (*do observe*), occurred twenty-six times in the Qur'an.¹¹ Similarly, Allah ordered the believers to make observation, like *yanzuruna* (يَنْظُرُونَ) (*they observe*), is also used nineteen times in the Qur'an,¹² *Awa lam yatafakkaruna* (أَوَلَمْ يَتَفَكَّرُوا) (*why do not they contemplate*), is also used three times in the Qur'an,¹³ *Yatafakkaruna* (يَتَفَكَّرُونَ) (*they contemplate*), is also used eleven times in the Qur'an.¹⁴ Allah declared Himself as *yudabbir al-amr* (يُدَبِّرُ الْأَمْرَ) (*He devises strategies*), and it is also used four times in the Qur'an.¹⁵

Similarly, the words *afala yatadabbaruna* (أَفَلَا يَتَدَبَّرُونَ) (*why don't they extract solution*), is also used two times in the Qur'an.¹⁶ The words *afala tubsirun* (أَفَلَا تُبْصِرُونَ) (*why don't you utilize vision*), is also used three times in the Qur'an.¹⁷

These orders of making observation (أَنْظُرْ), thinking and contemplation (تَفَكَّرْ), utilization of vision (بَصْر) and extraction of solution of influence (دَبْر) and conclusion of result (عَبْرَة) occurred in the Qur'an proved the basic sources for development of human thinking and basic steps and ingredients of scientific method. These Qur'anic orders compelled the human intellect to start thinking rationally and these verses also provided different steps of rational way of thinking.

It is the extreme emphasize on knowledge, thinking, vision and rationality in Islam. Theses Quranic teaching have developed raw human thinking to the level of scientific method and which

resulted in scientific revolution. Before Islam, no discipline of knowledge was in position to be called a discipline of science in the scientific sense. Hence the Quran provided the basis and the ingredient steps and scientific thinking which became the sources of invention of numeral and number system.

Mathematical Information & Concepts in Quranic Teachings and Inducement for Mathematical Research

Besides Quranic appreciation of knowledge and provision of basic steps of scientific level of thinking, some concepts of mathematical sciences are also present in Quranic verses which may be categorized as follows;





- i. Direct information and concepts of mathematical science given in Qur'anic teachings like numbering, counting, mathematical operation and properties, number system, sequence and series etc.
- ii. Indirect temptation of knowledge of mathematics, and use of mathematics in various aspects of religious rituals and duties in Islam like determination of time of salat (مواقيت), direction of Qibla, calculation of zakat, khiraj, 'ushr, law of inheritance (علم الفرائض), determination of calendar for Hajj, Ramadan etc.

Mathematic before revelation of Quran

The concepts of number system and counting, etc given in the Qur'an seem to be of less innovative value when we compare them with present number system but the importance of mathematical concepts given in Qur'an can be well determined when we see the status of number system at the time of advent of Islam. Before Islamic civilization, early Egyptian, Greek, Babylonian, Roman and Chinese civilizations used number system of their times. Although presentation of shapes or pictures seems awkward in research papers but some times pictures speak louder than words. So we are giving pictures or shapes of numeral digits and number systems of earlier civilizations for better and honest understanding of contribution of the Muslims to numeral and number system.

Egyptian Numerals and Number System

Early Egyptian (about 3400 BC) number system was heiroglyphic and different heiroglyphic symbols were used to represent numerals and units like one ten, one hundred, one thousand, etc like:

1		a vertical staff,
10		a heel bone,
10 ²		a scroll,
10 ³		a lotus flower,
10 ⁴		a pointing finger,
10 ⁵		a burbot fish,
10 ⁶		a man in astonishment


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The Egyptian hieroglyphic numerals were not in proper scientific form in both symbolic and conceptual aspects. There was no proper system of numbering and counting. The hieroglyphic numerals were very difficult in writing and understanding besides lackness of concepts of continuity, base, position value and proper symbols for numerals.¹⁹ The numbers were represented by hieroglyphic symbols and not by proper mathematical form. Even the large numbers were written very complicated form, e.g.,

Example;  = 13,545

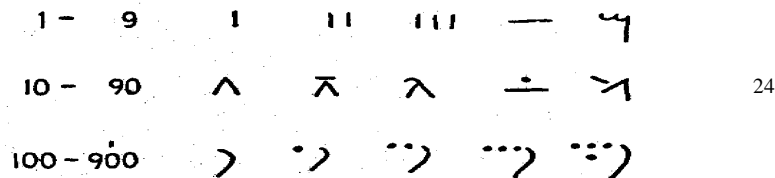
20

13015

= 

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Latterly, Egyptian hieroglyphic numerals were developed into two forms, that is, hierotic and demotic.²² In the herotic forms numerals from one to nine were denoted by symbols and multiple of 10 to 90, 100 to 990 and so on, at their individual sign but all numbers could not be represented as compact combination of such symbols.²³ Like:



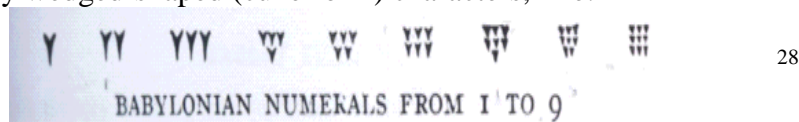
Hence we have seen that neither hieroglyphic nor hierotic form of Egyptian numerals were suitable and useful for development of number system. Their hieroglyphic nature is witness of their poor scientific nature.²⁵ Besides these, other deficiencies like shape of numerals and numbers were hazardous to the formation of proper number system in early Egypt. Hieroglyphic numerals were not justified to be called numerals in mathematical sense.²⁶ The hieratic numerals were also not with proper scientific concepts and symbols, like:



The mathematical operations like additions and multiplications were very difficult, puzzling and lengthy in the system.

Babylonian Numerals and Mathematical Concepts

In Babylonian civilizations, numerals from 1 to 9 were represented by wedged-shaped (cuneiform) characters, like:



The Babylonian numerals and number system were not properly based on simple number. It was partially based on 10 and partially on 60.²⁹ The representation of number was also complicated and unscientific in this system, like:



The shape of numerals was totally unscientific and illogical without any proper formula.³¹ The mathematical operation were not easily possible in this system.³²

Greek Numerals and Mathematical Concepts

The Greek used number system called attic, herodic or acrophonic in third century BC. The symbols for numerals come from the first letter of number names or symbols come from the observation of the word used for number,³³ like:

Ι	ΙΙ	ΙΙΙ	ΙΙΙΙ	Ϟ	ϟ	Ϡ	ϡ
1	2	3	4	5	6	7	8
1 - 10 in Greek acrophonic number							

Ϟ	Δ	Η	Χ
Pente	Deka	Hekaton	Khilioi
Πεντε	Δεκα	Ηεκατον	Χιλιοι
5	10	100	1000

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Lately, Greek used ionic or alphabetical system in which numerals were based on given values to the letters of the alphabets,³⁵ like:

<i>Units</i>	A	B	Γ	Δ	E	F	Z	H
	1	2	3	4	5	6	7	8
<i>Tens</i>	Ι	K	Λ	M	N	Ξ	Ο	Π
	10	20	30	40	50	60	70	80
<i>Hundreds</i>	P	Ξ	T	Υ	Φ	Χ	Ψ	Ω
	100	200	300	400	500	600	700	800

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The bar was used to distinguish numerals from alphabets and numbers were formed by additive principal. Hence it is clear from above forms that Greek numerals lacked the concept of position-value, even the shapes representing digits or numerals were not

proper and scientific. The Greek numerals were not scientific, complete, comprehensive and not occurring in logical manners.³⁷ There were many shortcomings in Greek numerals like no proper numeration scheme, non-continuity of numbers and representation of higher numbers were presented in Greek alphabetical system which resulted to complication of arithmetical operation and not helpful to computation.³⁸ Consequently, resulted to restriction of development of number system.³⁹

Roman Numerals and Mathematical Concepts

Roman numerals were represented by special symbols but these were not fit for arithmetical operation particularly, for multiplication and division. The calculations were laborious and clumsy,⁴⁰ like:

Example: MDCCCXXVII = 1,827.

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It is quite obvious that for simplest arithmetical problem, roman numerals called for enormous time and labour.⁴² Moreover, the Roman numerals have many deficiencies as discussed for Greek numerals, due to which no proper scientific number system could be evolved by the Romans.

Chinese, Japanese and Indian Numerals and Mathematical Concepts

The Chinese and Japanese numerals also contained many deficiencies, like:

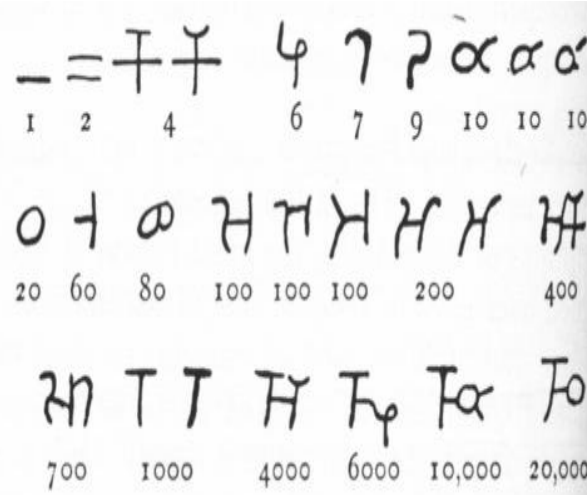
CHINESE-JAPANESE NUMERALS		
1 一	10 十	Example: 3468
2 二	100 百	三
3 三	1000 千	千
4 四		四
5 五		百
6 六		六
7 七		十
8 八		八
9 九		

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Similarly, in India different forms of numerals were adopted in different times and territories.⁴⁴

The earliest known found in King Asoka inscription in third century BC but the symbols were not uniform.⁴⁵



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Other forms for Indian numerals transmitted to India from the Arabia after Muslim's conquest. Hence we have seen that no proper symbols or shapes, concepts of continuous number, place value and base were found before the revelation of Qur'an. Even after first nine numerals no continuous form of numbering was present in the world. Shapes of digits or numerals were either alphabetic or hieroglyphic or iotic and not scientific before Islam. That is why number system and mathematics could not be developed to the scientific level before Islam. The numerals and number system before Islam were different in various civilizations and were not simplified, unified, proper and scientific in representation. Hence, calculations, if not impossible, were difficult, lengthy, complicated, confusing, laborious, non-understandable, clumsy, time consuming and unscientific before Islamic civilization.

Mathematical Concepts in the Qur'an

The mathematical information and mathematical concepts are present in numbers of Quranic verses. Many delicate and deep mathematical facts are present in these mathematical concepts. These mathematical concepts incited many Muslim scholars to inquire into the mathematics. These mathematical concepts consist of Numbering, Reckoning and Counting, Number System,

Fraction, Continuity of System, Measurement of Time Interval and calendar; mathematical operations like addition, subtraction, multiplication, division; mathematical properties, proportions, equality, continuity, infinity, exact measurement or line and time measurement, geometric sequence etc. These concepts are very important. They have status of foundational stones in the discipline of mathematics.

Numbering and Calculation of Calendar

The art of numbering and counting is present in many verses of the Holy Quran in appreciative manner, e.g. The Holy Quran describes about numbering and calculation by telling about the men of caves (Ashab Kahf) in Al-Kahf, 18:9-12

أَمْ حَسِبْتَ أَنَّ أَصْحَابَ الْكَهْفِ وَالرِّمِّمِ كَانُوا مِنْ آيَاتِنَا عَجَبًا ۝ إِذْ أَوَى الْفِتْيَةُ إِلَى الْكَهْفِ فَقَالُوا رَبَّنَا آتِنَا مِنْ لَدُنْكَ رَحْمَةً وَهَيِّئْ لَنَا مِنْ أَمْرِنَا رَشَدًا ۝ فَضَرَبْنَا عَلَى آذَانِهِمْ فِي الْكَهْفِ سِنِينَ عَدَدًا ۝ ثُمَّ بَعَثْنَاهُمْ لِنَعْلَمَ أَيُّ الْحِزْبَيْنِ أَحْصَى لِمَا لَبِثُوا أَمَدًا ۝

*"Do you consider 'The sleeper of the cave' and the 'Bearers of the inscription' to be among our strange signs? When those few young men took refugee in the cave, they said 'Our Lord! have a special mercy on us from thyself and guide us out of our ordeal aright.' Accordingly, we lulled them to a deep sleep in the cave for a number of years. Then we raised them up in order to test which of them were able to calculate correctly the period of their stay there."*⁴⁷

A similar concept is again given in same Sura (Al-Kahf, 18:19):

وَكَذَلِكَ بَعَثْنَاهُمْ لِنَتَسَاءَلُوا بَيْنَهُمْ قَالَ قَائِلٌ مِنْهُمْ كَمْ لَبِثْتُمْ قَالُوا لَبِثْنَا يَوْمًا أَوْ بَعْضَ يَوْمٍ قَالُوا رَبُّكُمْ أَعْلَمُ بِمَا لَبِثْتُمْ فَابْعَثُوا أَحَدَكُمْ بِوَرِقِكُمْ هَذِهِ إِلَى الْمَدِينَةِ فَلْيَنْظُرْ أَيُّهَا أَزْكَى طَعَامًا فَلْيَأْتِكُمْ بِرِزْقٍ مِنْهُ وَلْيَتَلَطَّفْ وَلَا يُشْعِرَنَّ بِكُمْ أَحَدًا ۝

"And in the same miraculous way we raised them up again so that they may ask one another. One of them asked, 'Well, how long have you remained in this condition?' The other answered 'may-be, we have remained so far a day or a part of day.' Then they declared, 'Your Lord knows best how long we have remained in this condition.'" ⁴⁸

In above mentioned verses, Quran, by discussing about men of cave, describes the test of correct calculation of period of their stay in the cave as an object of this event. In this verse, as the concept of numbering is given, as well, by describing the calculation of period of time, the concept of calculation is also given. For the determination of period of time (i.e. length of time interval) i.e. days, months and years, the sufficient knowledge of Taqweem (Calendar) is required. And for preparation and determination of calendar, the knowledge of earth, sun, moon, their orbits, their relative motion and movement in their orbits is required, besides the knowledge of mathematics, trigonometry, spherical trigonometry and astronomy.

In the second verse, by mentioning "A day or a part of a day" and by dividing a day into parts, one has been incited to deep research in determination of calendar. The division of a day into different parts also indicates about integral numbers and decimal fraction etc. Hence in above two verses, the concepts of calculation and numbering are given and standard of calculation and its accuracy has been declared as an object.

Number Theory and Fraction

Now after analyzing the position of number system before Islam, we can well analyze the mathematical and numeral concepts given in Qur'an like some signals about number theory are also given in this verse: (Al-Kahf, 18:22):

سَيَقُولُونَ ثَلَاثَةٌ رَّابِعُهُمْ كَلْبُهُمْ وَيَقُولُونَ خَمْسَةٌ سَادِسُهُمْ كَلْبُهُمْ رَجْمًا بِالْغَيْبِ
وَيَقُولُونَ سَبْعَةٌ وَثَامِنُهُمْ كَلْبُهُمْ قُلْ رَبِّي أَعْلَمُ بِعَدَّتِهِمْ مَا يَعْلَمُهُمْ إِلَّا قَلِيلٌ فَلَا تُمَارِ
فِيهِمْ إِلَّا مِرَاءً ظَاهِرًا وَلَا تَسْتَفْتِ فِيهِمْ مِنْهُمْ أَحَدًا

*"Some will say, 'They were three and the fourth was their dog,' and some other will say, 'They were five and sixth was their dog.' These were mere irrelevant guesses. There are still others who say, 'They were seven and eighth was their dog.' Say, 'My Lord knows best how many they were.' There are a few people only who know their correct numbers."*⁴⁹

This verse contains many mathematical facts. It contains concept of probability besides the concepts, of numbering, counting, their characteristics and kinds (even numbers and their fractions - fourth, sixth, eights and odd numbers, i.e. three, five,

seven etc.), and formation of number system. By saying, "There are a few people only who know their correct number," the Muslims are incited to work on number system and number theory, and to acquire knowledge of this discipline of science. As a result of this Quranic inducement, Muhammad bin Musa Al-Khawarizmi worked on formation and properties of number system,⁵⁰ and the scholars of Ikhwan-us-Safa (إخوان الصفاء) also discussed the characteristic of numbers philosophically.⁵¹ Khushyar bin Liban also worked on the topic by writing *Kitab fi Usul-il-Hisab-il-Hindi* (كتاب في أصول الحساب الهندي).⁵² Latterly, Al-Uqlidisi worked on the number system in his book *Kitab-ul-Fasul fil Hisab* (كتاب الفصول في الحساب).⁵³

The concept of number is also given in Quranic verses revealed about earth, heavens etc. as follows: (Al-Mu'minun, 23:86):

قُلْ مَنْ رَبُّ السَّمَاوَاتِ السَّبْعِ وَرَبُّ الْعَرْشِ الْعَظِيمِ

"Say, "Who is the Lord of the seven heavens."⁵⁴

The concept of numbering in perspective of number of heavens is given in verses: (Al-Baqra, 2:29), (Al-Talaq, 65:12), (Al-Nuh, 71:15). The concept of numbering is also present in perspective of creation of universe like: (Al-A'raf, 7:54)

إِنَّ رَبَّكُمُ اللَّهُ الَّذِي خَلَقَ السَّمَاوَاتِ وَالْأَرْضَ فِي سِتَّةِ أَيَّامٍ.

"Our Guardian - Lord is Allah, who created the heavens and earth in six days."⁵⁵

The same concept is given in Quranic verses, (Al-Hud, 11:7), (Al-Furqan, 25:59), (Al-Sajda, 32:4), (Qaf, 50:38).

Exact Measurement

The important property of numbers i.e. quantitative exact measurement, is also given in Quranic verses, e.g. (Al-Baqara, 2:80):

وَقَالُوا لَنْ تَمَسَّنَا النَّارُ إِلَّا أَيَّامًا مَعْدُودَةً.

And they say, "The fire shall not touch us but for a few numbered days."⁵⁶

(Al-Baqra 2:203)

وَاذْكُرُوا اللَّهَ فِي أَيَّامٍ مَعْدُودَاتٍ.

Celebrate the praises of Allah during the appointed days.⁵⁷

Counting

The concept of counting is in the verse about stay of men on earth as it describes:

(Al-Mu'minun, 23:112-114):

قَالَ كَمْ لَبِثْتُمْ فِي الْأَرْضِ عَدَدَ سِنِينَ ۚ قَالُوا لَبِثْنَا يَوْمًا أَوْ بَعْضَ يَوْمٍ فَاسْأَلِ الْعَادِينَ ۚ قَالَ إِنَّ لَبِثْتُمْ إِلَّا قَلِيلًا لَوْ أَنَّكُمْ كُنْتُمْ تَعْلَمُونَ ۚ

"Then Allah will inquire from them, "For how many years did you stay on the earth." They will say, "We stayed there for a day or for a part of a day. You may inquire this from those who kept account."⁵⁸

Here number of years (عدد السنين) has been asked by men of caves. But they replied that this question of number of years (duration of stay) may be asked by the persons who kept account. Hence there should be persons in society who should keep account. But the science of accounting must be known to the persons who will keep account of the calendar. The same concept is given in another verse: (Al-Maryam 19:94-95).

Numbering and Infinity

The concept of counting (numbering) and its properties are also present in this verse. (Al-Nahl, 16:18) (Ibrahim, 14:34):

وَأِنْ تَعَدُّوا نِعْمَتَ اللَّهِ لَا تَحْصُوهَا.

"If you try to reckon up Allah's blessings, you can not count them."⁵⁹

This Quranic verse yields concept of numbering (counting) as well as infinite concept of numbers. Or more generally this Quranic verse contains the continuity of number system. The concept of accounting is also given in perspective of life of heavens and earth as Quran describes. (Maryam 19:94)

لَقَدْ أَحْصَاهُمْ وَعَدَّهُمْ عَدًّا ۚ

"He does take an account of them (all), and hath numbered them (all) exactly."⁶⁰

These mathematical (astronomical) facts, given in Quran, got the attention of Muslim scientists. And they devoted their lives in research of such topics and consequently contributed a lot to the humanity. These concepts guided the intellect of Muslim scholars who worked on the number system and accounting.

Mathematical Operations

The concepts of basic mathematical operations (like, addition, division, multiplication, subtraction, etc.) and properties (like, equality or balance, proportion etc.) are given in Quran in

very deep philosophical manner in different perspectives, e.g.

Division and Fraction

The concept of division of a number into fraction is given in Sura (Al-Kahf, 18:19):

قَالُوا لَبِثْنَا يَوْمًا أَوْ بَعْضَ يَوْمٍ.

*"They said, "We have stayed a day or a part of day."*⁶¹

Balance and Proportion

The concept of balance and proportions are also given in Quran e.g. Quran says: (Al-Hijr, 15:19):

وَأَنْبَتْنَا فِيهَا مِنْ كُلِّ شَيْءٍ مَوْزُونٍ

*"And produced there in all kinds of things in due ballance."*⁶²

Balance and proportion are basic character of arithmetic and algebra.

Multiplication

The concept of multiplication is given in Sura-al-Baqra, in a astonishing way, along with many other mathematical and algebraic concepts. As Quran describes: (Al-Baqara 2:261):

مَثَلُ الَّذِينَ يُنْفِقُونَ أَمْوَالَهُمْ فِي سَبِيلِ اللَّهِ كَمَثَلِ حَبَّةٍ أَنْبَتَتْ سَبْعَ سَنَابِلَ فِي كُلِّ سُنبُلَةٍ مِئَةُ حَبَّةٍ وَاللَّهُ يُضَاعِفُ لِمَنْ يَشَاءُ وَاللَّهُ وَاسِعٌ عَلِيمٌ

*"The parable of those who spend their substance in the way of Allah is that of a grain of Corn: It growth seven ears, and each ear hath a hundred grains. Allah gives manifold increase to whom. He pleases: And Allah careth for all and He knoweth all things."*⁶³

There is very deep concept of multiplication in this verse. As it is described that each grain has seven ears and each ear has hundred grains. The number of grains can be determined by multiplication.

Geometric Sequence and base 10

Some basic signals are given in this verse about the formation of geometric sequence and series. If each grain, after cultivation, has seven ears, and each ear has hundred grains. Then after first term of cultivation, the number of grain produced will be as:

1. Ist Term $7 \times 100 = 700$ or 7×10^2

Then if 700 grains are cultivated, the produced number of grains, as given in this verse, will be:

$700 = 7 \times 100$. After 2nd Term $7 \times 100 \times 7 \times 100 = 490000$

$$\text{Result} = 490000 \text{ or } 7^2 \times 10^4$$

If the grains produced after 2nd term of cultivations are cultivated third time, the result will be: $490000 = 7^2 \times 10^4$ After 3rd Term $7^2 \times 10^4 \times 7 \times 100$

$$\text{ie } 7^3 \times 10^6$$

The words, "Allah giveth manifold increases" indicates towards continuity of process up to any limit or infinite terms. Hence if we write the result of first five terms of cultivation, it will be:

$$1, 7 \times 10^2, 7^2 \times 10^4, 7^3 \times 10^6, 7^4 \times 10^8$$

If we proceed it to infinite terms it will be:

$$1, 7 \times 10^2, 7^2 \times 10^4, 7^3 \times 10^6, 7^4 \times 10^8, \dots$$

The sequence is increasing or decreasing to the right or left respectively by the ratio 7×10^2 , hence, by definition, it is geometric sequence which has prime importance in arithmetic, astronomy, algebra, and applied mathematics. If we add the terms of sequence it will be geometric series and we can find the net sum of series or any missing term of sequence or series, if we have three terms or two terms and common ratio. A similar concept is again given in Quran: (Yusuf, 12:43). This verse also contains ratio as 7×10^2 which clearly indicates about base 10 of number system as ratio is always linked with base.

These and other such like, mathematical facts and realities and concepts influenced very deeply the minds and thoughts of Muslim scholars. These philosophical and mathematical concepts developed the attitudes of Muslim scholar towards mathematics and gave guide lines as contrary to any other religions of the world. The concepts of number system, numbering and counting, reckoning and operations like, addition, subtraction, multiplication, decimal fraction, kinds of numbers, their characteristics, etc. are of basic importance in the history of making of mathematics, and Quran incited the Muslims to work on such topics.

Mathematics before Islamic era was so raw and poor that its basic intity i.e. numerals digits were not with proper and logical shapes and the concepts, as we have seen in the above given shapes of numeral of early civilizations. That's why mathematics could not go beyond mere shepherd based counting in hundred years of history. The concepts of numeral and mathematics given in Quran incited and guided the human intellect for devoted work on mathematics. The religious requirements or determination of times of prayers, calculation of Zakat, determination of calendar of

Hajj and Ramzan, determination of direction of Qibla and Hajj routes; and solutions of problems of inheritance⁶⁴ compelled the Muslims intellect for finding out mathematical facts, concepts and formulas which all lead to invention of numeral and number system, arithmetic, geometry, trigonometry, spherical trigonometry, and algebra. Muhammad bin Musa al-Khawarizmi, the head of bait ul Hikma, wrote first book on the subject of numeral system called "Kitab ul Hisab"⁶⁵, containing Arabic numerals and decimal system.⁶⁶ Its several Latin translations exist in the West⁶⁷ (like Cambridge University manuscript), and its most popular Latin version is *alchwarizmi's alcorismus* edited by Kurt Vogel.⁶⁸ Alkhawarizami wrote other two books on the subject called "Kitab ul Jama wa Tafreeq"⁶⁹ and "Kitab Jama fi he bayanul Hisab wal Hindsa wal Musiqi wal Falak" whose Latin translation named *Liber ysagogarun Alchoarismi* exists⁷⁰. It was also published by Negal, Curts, and Haskin⁷¹.

Alkhawarizami's books described numeral digit 1, 2, 3, 4, 5, 6, 7, 8, 9 first time in the history of mankind.⁷² These numerals are shape or figured based numeral or more generally, they have been shaped or made by their angle specific value like digit 2 contains 2 angles. Alkhawarizami also invented zero⁷³, the greatest concept of mathematics. He also established his number system with concept of place – value, with base 10 which is being used commonly now a days. He also established sexagesimal (base 60) which was invented for spherical calculation of calendar.⁷⁴

Hence it is the blessing of Quran and Islam which leaded and inspired the human intellect for mathematical research and some of its teachings became source of mathematical inventions. The invention or creation of present numeral and number system is indebted to Quranic teachings.

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